the matrix problems given in the text. How to get software over the Internet and the author's MATLAB toolkit for test matrices are also discussed in the appendices.

There are over 200 problems in the volume and they resonate with the text very well. Except for the "research problems," solutions to the exercises can be found in a 50-page appendix. The book could be used as a text for advanced graduate-level courses in matrix computations. However, the main role that the book will assume in the coming years will be as a reference and as a companion text in the classroom. Wilkinson's *Algebraic Eigenvalue Problem* played a similar role in the 1970s and 1980s and I bet Higham's book will prove to be equally valuable in the long run.

The volume is laced with great quotations and my favorite is due to Beresford Parlett:

> One of the major difficulties in a practical [error] analysis is that of description. An ounce of analysis follows a pound of preparation.

No numerical analyst can change that ratio. But what Higham has shown is that this quote "scales up." With tons of preparation Higham has given us a hundred-weight of analysis—enough to keep the field on solid foundation for years to come.

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21[65N06, 65-04]—*Algorithms for elliptic problems: Efficient sequential and parallel solvers*, by Marián Vajteršic, Mathematics and Its Applications (East European Series), Vol. 58, Kluwer, Dordrecht, 1993, xviii + 292 pp., $24\frac{1}{2}$ cm, \$152.00/Dfl.240

This book presents a survey of fast numerical methods for elliptic partial differential equations. Chapters 1 and 2, forming the first of the two parts of the book, examine methods designed for sequential computers. Chapters 3 through 6, forming the second part of the book, consider extensions to parallel computers. While the title of the book implies a somewhat general treatment of the subject, the author deals mainly with the case of finite difference discretizations of linear second- and fourth-order elliptic equations with Dirichlet boundary conditions on the unit square in two dimensions. However, the treatment of this restricted set of topics is quite thorough. Each chapter begins with a resonably complete review of the related literature, as well as an entertaining historical overview. An extensive bibliography follows each chapter.

In Chapters 1 and 2, fast sequential methods for the Poisson and biharmonic equations are examined. Chapter 1 begins with direct methods based on the FFT, cyclic reduction, and marching algorithms. Suitable modifications for handling various boundary conditions are then discussed. The survey then moves to iterative methods, such as classical relaxation and multigrid methods. While the unit square domain is considered for most of the book, the first chapter ends with a discussion of the treatment of L-shaped, octagonal, and circular domains, again remaining in the finite-difference setting. Chapter 2 begins with direct methods for the biharmonic equation and concludes with methods for the biharmonic eigenvalue problem. The presentation in Chapter 2 includes direct methods (the Buzbee-Dorr algorithm, Bjorstad's algorithm, and Golub's algorithm), and methods based on operator splitting.

The focus of Chapters 3 through 5 is on the extension of the methods of Chapters 1 and 2 to parallel computers. A discussion of the basic ideas of solving elliptic equations in parallel is presented in Chapter 3. Parallel versions of marching algorithms, cyclic reduction, and domain decomposition algorithms for the Poisson equation are examined in detail, followed by a discussion of the parallel implementation of classical relaxation iterative methods. The biharmonic equation is the subject which concludes Chapter 3; the presentation includes the extension of parallel methods for the Poisson equation to the biharmonic case. In Chapter 4, the implementation of the methods of Chapter 3 on specific computers is examined, including implementation on the ICL DAP, the CDD Star-100, the Cray-1, the EGPA, the Connection Machine, and the MasPar computer. While Chapter 4 is the most practical chapter of the book, and gives the most meaningful performance indications for the various parallel algorithms considered, it also dates the book due to the rapid evolution of computer hardware. Chapter 5 examines the parallel implementation of a single algorithm, namely the multigrid method. Since this algorithm has such favorable complexity properties, and yet is somewhat complex to implement in parallel, it is no surprise that the author devotes more than forty pages to this topic. The chapter includes discussion of basic principles, SIMD versus MIMD algorithms, hypercube topologies, and complexity models for various hardware topologies. The chapter concludes with some numerical experiments on a Cray X-MP and other machines.

Chapter 6, the final chapter, is in some ways the most interesting chapter of the book. The topic is the implementation, in hardware, of some of the parallel algorithms (e.g., cyclic reduction and multigrid) presented earlier in the book using VLSI technology. The presentation is complete with VLSI schematics for various core computational kernels. Algorithms are constructed by piecing the schematics together, much as they would be laid out in silicon. The chapter concludes with a discussion of the biharmonic operator, complete with the VLSI schematic layout of a capacitance matrix solver and a multigrid-based solver.

While the first two chapters contain complete complexity statements for all of the methods presented, notable omissions throughout the rest of the book are theoretical performance models appropriate for parallel computers, which would provide a framework for meaningful performance analysis and comparisons of the various methods. (An exception is the excellent discussion in Chapter 5.) Rather than presenting these performance models, the author gives timings on various computers (many of which are now extinct), which unfortunately dates the book somewhat. However, the author makes it clear in the Introduction that the omission of complex theoretical models was intentional for readability. The book is otherwise thorough, well-written and contains few errors. The notation is simple and easy to follow; a list of the symbols used appears at the beginning of the book. This book could be used in a course on parallel numerical methods for model elliptic equations in two dimensions.

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